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course W and W' might be so related that this would be true, but in general they are not. To see at which point there is limiting friction, we might proceed as follows: Solve on the supposition that limiting friction exists between sphere and box, and not between sphere and wall; then solve with the opposite assumption; one of these results will be found smaller than the other, I think it is the latter; we infer then that limiting friction must exist at wall and not at box, and hence that the latter result is the correct one. W. H. DRANE.

88. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics, Decorah Institute, Decorah, Ia.

Show that the equation to the trajectory is

$$y=x\tan\alpha-\frac{gx^2}{2v^2\cos^2\alpha},$$

and that v and a can be varied at pleasure, the projectile can in general be made to traverse any two given points in the same vertical plane with the point of projection. [Ex. 83, page 244, Deschanel's *Natural Philosophy*, Part I.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.; B. F. SINE, Principal of Normal School, Rock Enon Bridge, W. Va.; ELMER SCHUYLER, Reading, Pa.; and the PROPOSER.

Let v=velocity of projection, α=angle of elevation, t=time, (x, y) the coördinates of the point in its path at the time t.

$\therefore$ $x=vt\cos\alpha$=horizontal motion. $y=vt\sin\alpha-\frac{1}{2}gt^2$=vertical motion.

Eliminating t, we get at once,

$$y=x\tan\alpha-\frac{gx^2}{2v^2\cos^2\alpha}\ldots\ldots(1).$$

(1) is true no matter what be the values of v and α.

Let (m, n), (b, c) be the coördinates of two points. Then from (1) we get

$$n=m\tan\alpha-\frac{gm^2}{2v^2\cos^2\alpha},\quad c=b\tan\alpha-\frac{gb^2}{2v^2\cos^2\alpha}.$$

$$\therefore\ \alpha=\tan^{-1}\left(\frac{nb^2-m^2c}{mb^2-m^2b}\right),\quad v^2=\frac{g[(mb^2-m^2b)^2+(nb^2-m^2c)^2]}{2(mb^2-m^2b)(bn-mc)}.$$

These values of α and v will cause the trajectory to pass through the two given points.

89. Proposed by GUY B. COLLIER, Schenectady, N. Y.

Assuming that the Northern Pacific R. R. tracks between Fargo and Bismark (North Dakota) to lie on the 47th parallel of latitude; also that the Limited Express weighs 300 tons, and that a speed of 60 miles per hour is maintained between the two places · find the difference between the vertical pressures on the rails of the Express east and the express west.

I. Solution by WALTER H. DRANE, Graduate Student, Harvard University, Cambridge, Mass.

Centrifugal force$=mv^2/\rho_1$.

Taking $g=32$ we have $m=18750$ for mass of train.

For 47th parallel of latitude $\rho=14403840$ feet, and $v=1048.89$ feet per second is velocity of a point at that parallel.

The velocity of the train on the surface of the earth is 88 feet per second.

$\therefore$ $1048.89-88=960.89$ feet is velocity in space of train going west.

$1048.89+88=1136.89$ feet is velocity in space of train going east.

For train going west we have for centrifugal force,

$$\frac{mv^2}{\rho}=\frac{18750(960.89)^2}{14403840}=1201.1059 \text{ pounds.}$$

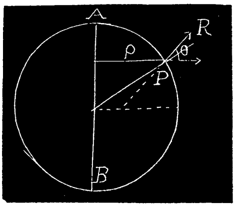
For train going east,

$$\frac{mv^2}{\rho}=\frac{18750(1136.89)^2}{14403840}=1682.1923 \text{ pounds.}$$

$1682.1923-1201.1059=481.0864$ pounds, amount by which the decrease in weight of train going east exceeds the decrease in weight of train going west.

If by vertical pressure is meant that towards center of circle of latitude, the above is the excess in pressure of train going west over that of train going east.

If the pressure is understood to act towards the center of the earth the difference is $(481.0864)\cos 47^\circ=328.6312$ pounds.



If however, the difference in the weight of the two trains is required, we must proceed as follows :

We have assumed the earth to be a perfect sphere. On account of the motion of the earth the apparent line of weight is slightly deflected from the vertical. Let PR be the line of action of apparent weight, $R=mg$ at a point P on the earth's surface. Let G be the value gravity would have if the earth were still, and then the force along PO will be mG. Let ϕ be the complement of the latitude of P. Resolving along CP and perpendicular to it we get

$$\frac{mv^2}{\rho}=mG\sin\phi-mg\cos\theta\ldots\ldots(1).$$

$$mg\sin\theta=mG\cos\phi\ldots\ldots(2).$$

Eliminating θ from (1) and (2) and solving for G gives

$$G=\frac{v^2\sin\phi}{\rho}\pm\sqrt{g^2-\frac{v^4}{\rho^2}\cos^2\phi}.$$

Substituting values for the case in hand we find $G=32.05209$.

From (2),

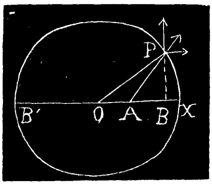
$$\sin\theta=\frac{G\cos\varphi}{g}=\frac{32.05209\times .731}{32}=.73219.$$

$\therefore\ \theta=47^\circ\ 4'\ 15''$.

$\therefore$ Difference in apparent weight=481.0864cos(47° 4′ 15″)=327.6583 lbs.

II Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

This problem depends on the velocities in space of the Express east and the Express west.



In the figure, let P be the point of the train on the 47th parallel, O the center of the earth, AP the normal at P, φ the angle PO makes with the equatorial diameter $B'x$, θ the angle the normal makes with the same line, (θ is the latitude of P).

Let $OB=\rho$, $OP=r$, 300 tons$=W$, f=centrifugal force in the direction AP (vertical direction), g=gravity on 47th parallel, G=gravity at equator. a=6377377 meters=20923536 feet=equatorial radius, e=ellipticity of the earth.

$\rho=r\cos\varphi=a\cos\theta/\sqrt{(1-e^2\sin^2\theta)}$.

Now $\theta=47^\circ$, $e^2=.006920928$. $\therefore\ \rho=4357445.45$ meters.

One day=86400 seconds.

$\therefore\ 2\pi\rho/86400=316.8831$ meters=1039.37 feet per second, the velocity of P due to the earth's rotation.

60 miles an hour=88 feet per second.

1039.37−88=951.37, the train's velocity in space going west.

1039.37+88=1127.37, the train's velocity in space going east.

$f=Wv^2/g\rho=Wv^2/gr\cos\varphi$.

$\therefore\ F=f\cos\theta=Wv^2\sqrt{(1-e^2\sin^2\theta)}/ag$.

Now $g=G(1+\frac{1}{4}e^2\sin^2\theta)$.* $G=32.2015235$ feet.

$\therefore$, $g=32.23130991$ feet per second. $\therefore\ F=.000000444v^2$ tons.

$\therefore\ F=.4018399$ tons going west. $F=.5643076$ tons going east.

$\therefore$ Difference=.1624657 tons=324.9354 pounds.

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*This expression for the true value of gravity in latitude θ is new to me. If any reader of the MONTHLY can tell me where to find it used previously, and by whom, I would be greatly pleased to know. I believe it to be new and unused before.

AVERAGE AND PROBABILITY.

76. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

In a given ellipse, the extremities of a focal chord are joined with the center. Find the average area of the angle thus formed.